LAGRANGE MULTIPLIER (LM) TEST FOR NONLINEARITY DETECTION IN STRUCTURAL EQUATION MODELING (SEM)

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Abstract. Linearity test is important step in statistical analysis. It will certify the statistical method, i.e. linear or nonlinear, which is employed in data analysis. The objective of this paper is to propose a Lagrange Multiplier (LM) test for nonlinearity detection in Structural Equation Model (SEM). This LM test is needed to validate the linear relation assumption between latent variables in standard SEM. Moreover, this LM test is also very crucial and needed before applying non-linear SEM, i.e. SEM with polynomial and interaction of relation between latent variables. In this paper, a simulation study is carried out to verify the effectiveness of this linearity test both in partial and simultaneous relation between latent variables. The results show that the LM test is an effective tool to detect nonlinearity relation between latent variables in nonlinear SEM. Furthermore, this LM test is also an effective tool to validate the linear relation assumption between latent variables in standard SEM.

Keywords: LM Test, Linearity Test, Standard SEM, Nonlinear SEM, Polynomial, Interaction.

1 Introduction

In last two decades the Structural Equation Model (SEM) has been widely used in social research. SEM is a model that represent a relation between latent variables which could not be measured directly. SEM become powerful because it enables the researchers to estimate the coefficient of linear model with controlling the bias effect in error measurement Kenny and Judd [1].

It is needed to develop an analysis that cover nonlinear relation between latent variables, specially in social science and behavior research Moosbrugger, Engel, Klein, Kelava [2]. Lee [3] clarify that nonlinear analysis is important to get a strong analysis and interpretation in order to find a suitable model particularly for a complex problems. From previous research, nonlinear effect is caused by interaction effect between predictors or quadratic effect. Interaction effect occur when relation between predictors and criterion become strong or weak because of the second predictors. The quadratic effect arise when the predictor interacted with itself Moosbrugger *et.al* [2].

Kenny and Judd [1] explain the simple thing of structural model of latent variables that is: it involve two estimation procedures. First it takes factor analysis and second it put the covariance between factors to multiple regression procedure. However there is an advantage of linear model estimation in multiple regression which is different with latent variable model. This advantage of multiple regression is: the effect of nonlinear will be estimated in simple way.

In multiple regression, the assumption of relation between variables is a linear function. For instance, Y linear function of X_1 and X_2, thus X_1 and X_2 linear to Y, but it is possible to non-linear relation in the function if the relation X_1 to Y induce with X_2 or X_1 itself. The analysis in interaction effect put a cross product between predictors ($X_1\ X_2$) and quadratic effect will put X_1^2 atau X_2^2.

Some methods have been developing the estimation of interaction effect in SEM. Kenny & Judd ([1] proposed a model of interaction between latent variables (predictors) were estimated with cross product of indicators. In 1995 this method expanded by Jaccard & Wang which use multiple cross product of indicators. In 1996 Joreskog & Yang suggested a method with single cross product of indicators. Other alternative methods developed in 1998 by Johnsons then Joreskog in 2000 which suggested that the interaction analysis was directly estimated with cross product of factor scores of latent variables Porzio and Vitale [4]

One of the important step in nonlinear analysis is linearity test. This test will analyze both in relation between indicators and relation between latent variables. Porzio and Vitale [4] proposed the linearity test on SEM model by using graphical approach. This paper proposed a simulation study of linearity test in SEM with Lagrange Multiplier Type Test.

2 Linearity Test with Lagrange Multiplier Type

Lagrange Multiplier test (LM test) is an alternative of Ramsey's Test. Testing problems non-linear hypothesis such as $g(\theta) = 0$ are considered where g is a $p \times 1$ vector function define on θ. If the true value of θ under the null be θ^0 then $g(\theta^0) = 0$. The expand of Taylor series:

$$g(\theta) = g(\theta^0) + G(\bar{\theta})(\theta - \theta^0) \quad (1)$$

where $\bar{\theta}$ lies between θ and θ^0 and $G(.)$ is the first derivative matrix of g, for the null, θ approach θ^0 so $G(\bar{\theta}) \rightarrow G(\theta^0) = G$ and the restriction of linear hypothesis is $G\theta = G\theta^0$

Lagrange Multiplier test is derive from a constrained maximization principle. The maximization of the log-likelihood subject to the constraint that θ=θ^0. Let H be a Lagrangian:

$$H = L(\theta, y) - \lambda'(\theta - \theta^0) \quad (2)$$

the first order condition are:

(3)

$$\frac{dL}{d\theta} = \lambda; \quad \theta = \theta^0$$

So $\lambda = s(\theta^0, y)$. Assuming a central limit theorem then:

$$\xi_{LM} = s'(\theta^0, y)\mathcal{F}^{-1}(\theta^0)s(\theta^0, y)/T \tag{4}$$

Will have a limiting χ^2 distribution with k degrees of freedom under the null, where T is vector random of samples, and $\mathcal{F}$ is Fisher's information.

The procedure of LM test is if there are two function with two predictors, for instance in linear or cubic then the first function will be a restriction of the second function. An example: if there is a linear function

$$Y_i = \lambda_1 + \lambda_2 X_{1i} + \lambda_3 X_{2i} + u_i \tag{5}$$

and the next function in cubic is :

$$Y_i = \beta_1 + \beta_2 X_{1i} + \beta_3 X_{2i}^2 + \beta_4 X_{1i}^3 + u_i \tag{6}$$

The algorithm of LM test is :

1. Estimate the restriction regression with OLS of first equation (5) to obtain residual $\widehat{u_\iota}$.
2. If the second equations are the true regression, thus the residual which were obtained from the first equation should be related to cubed and squared output term that is X_i^2 and X_i^3.
3. thus the equation of $\widehat{u_\iota}$ which is obtained from the first step is :

$$\widehat{u_\iota} = \alpha_1 + \alpha_2 X_{1i} + \alpha_3 X_{2i}^2 + \alpha_4 X_{1i}^3 + v_i \tag{7}$$

   Where v_i is error of each equation in i
4. For a large sample (n), multiply n with R^2 then estimation from the equation (7) follow the chi-square distribution with df equal to sum of restriction,thus it will be rid of the model, therefore it can be written as:

$$nR^2 \sim X^2_{(\text{number of } restriction)} \tag{8}$$

5. If the chi-square value which is obtained from equation (8) more than critical value of significant level, thus the restriction regression is rejected, the other way if chi-square value less than critical value of significant level then it is accepted.

2.1 Terasvirta test

Terasvirta test is one of Lagrange Multiplier test which employed in nonlinearity analysis based on theory of neural network. Subanar and Suhartono[17], explained there is not a test can detect all nonlinear possibility, thus it is possible to need more than one test. But the result of a test could be a direction about a nonlinearity of the model. In order to understand of Terasvirta test, consider this nonlinear model :

$$y_t = \Phi(y'\omega_t) + \pi'\omega_t + u_t \tag{9}$$

where $u_t \backsim \text{nid}(0, \sigma^2)$, $\omega_t(1, \widetilde{\omega_t})$, $\omega_t = (y_{t-1}, \dots, y_{t-1})$ $\pi = (\pi_0, \pi_1, \dots \pi_p)$

$y = (y_0, \bar{y}')$ $\bar{y} = (y_1 \dots y_p)$. Set :

$$\Phi(y'\omega_t) = \theta_0 \psi(y'\omega_t) \tag{10}$$

where

$$\psi(y'\omega_t) = (1 + \exp(-y'\omega_t))^{-1} - {}^{1}\!/_{2} \tag{11}$$

then the hypothesis y_t is linear :

$$H_0 : \theta_0 = 0 \tag{12}$$

Consider equation (9) interpreted as nonlinear autoregressive model in which the intercept is $\pi_0 + \psi(y'\omega_t)$ as a time varying and changes smoothly from $\pi_0 - {}^{\theta_0}\!/_{2}$ to $\pi_0 + {}^{\theta_0}\!/_{2}$ w with $y'\omega_t$. considering that model (9) is a special case of the following neural network model with a single hidden layer:

$$y_t = \pi'\omega_t + \sum_{j=1}^{q} \theta_0 \left\{\psi(y'\omega_t) - {}^{1}\!/_{2}\right\} + u_t \tag{13}$$

From (9) and (10) thus the hypothesis that y_t is linear i.e. $y_t = \pi'\omega_t + u_t$. Within (13) $H_0 : \theta_{01} = \cdots = \theta_q = 0$ is called the linearity hypothesis of neural network test. Note that $\psi(0) = 0$ another possible null hypothesis of linearity is

$$H_0' : □ = 0 \tag{14}$$

against the alternative $□ \neq 0$. The simplest approximation is the first-order one. From (10), it is seen that

$$\frac{\partial}{\partial □} = \psi(y'\omega_t)|_{□=0} = \psi'(0)\omega_t \tag{15}$$

Thus the approximation $\theta_0 t_1(y'\omega_t) = \theta_0 \psi'(0) y'\omega_t$ merge with linear part of model (9), so that all the information about non-linearity is lost. This is another way of seeing that (9) with (10) and the linear autoregressive model of order p are locally equivalent alternative with respect to (13) Terasvirta, Fu & Granger [18]

3 Simulation Study

Simulation study performed for analyzing the implementation Lagrange Multiplier type test on non linearity test of SEM model. The simulation will carry out in three data groups. The first group is a group with variance covariance matrix without multicolinearity, the second group with multicolinearity about 0.5 and the third group with multicolinearity close to 1 (0.8). This simulation was done in simple way, engage three latent variable with each indicators. The indicators and variable were randomly generate in 100 data ($n = 100$) with normal multivariate and $\varepsilon \sim (0, 0.1)$. linearity test was performed in two step. First, the test for linearity between latent variable and each indicator and second, the linearity between latent variable and all indicators.

The model of this simulation was shown in this diagram:

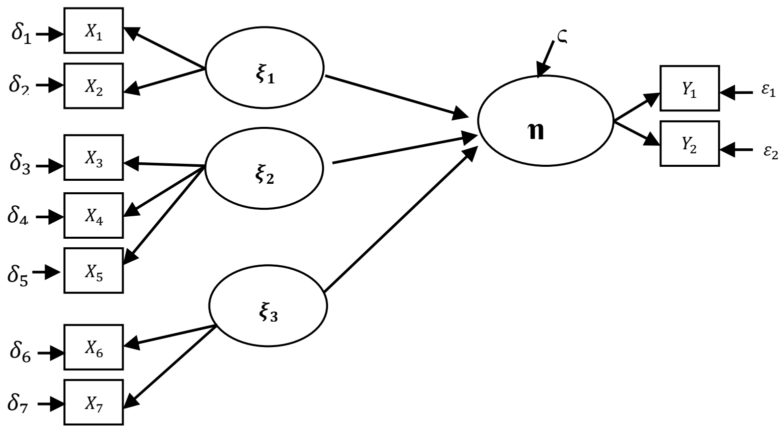

Figure 1 Structure of linear SEM

3.1 Linearity Test in Quadratic and Cubic Effect

The equation to investigate a quadratic effect in model is:

$$Y = X_1^2 - X_2^3 + X_3 + \varepsilon \quad (16)$$

From this simulation, the element of quadratic in model were obtained from the linearity of the first level quadratic which reach the level significant value. From table 1, the significant value of x_1^2=0.4177, x_2^3 =0.4103 dan $x_3 = 0.9915$, it means that X_3, x_1^2, x_2^3 become the elements of non linear relation between latent variable and indicators in $Y = X_1^2 - X_2^3 + X_3 + \varepsilon$

The simulation that employed variance covariance matrix with multicolinerity about 0.5, the linearity of each indicators completed these conditions: (a) for each indicator, the element of non linear will determine in the first structure of polynomial which reach the significant value. For instance, in table 2, p-value of x_1^2=0.5996 and p-value of x_1^3 =0.9825, both x_1^2 and x_1^3 reach the significant level. Because of x_1^2 is more simple structure of polynomial than x_1^3 thus x_1^2 will put as an element of non linear equation. (b) if all combination of indicators less than significant value, then the highest p-value will put as an element of non linear equation. (c) if all the p-value more than significant value, then the simplest structure of polynomial will put as an element of non linear equation

From simulation which multicoliniearity close to 1 (0.8), the combination of element quadratic in model, are more difficult to be recognize with significant value. It mean that linearity test for data with high multicolinearity should be analyzed in different way.

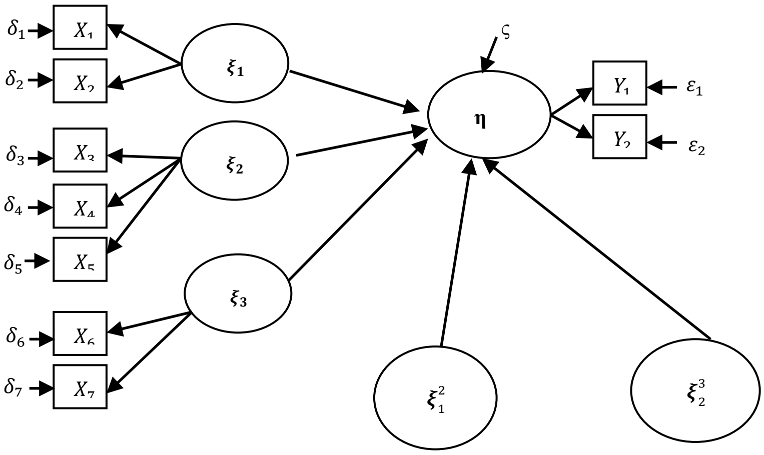

Figure 1 Structure of non-linear SEM with quadratic & cubic effect

Table 1. Result of non-linearity Test

Variable	Uncorrelated	Multicolinearity	
		0.5	**0.8**
$x_1 \rightarrow y$	0.0065	0.0002	$3.383e^{-08}$
$x_1^2 \rightarrow y$	0.4177	0.5996	0.04904
$x_1^3 \rightarrow y$	0.0013	0.9825	$2.600e^{-10}$
$x_2 \rightarrow y$	$<2.2e^{-16}$	$<2.2e^{-16}$	$<2.2e^{-16}$
$x_2^2 \rightarrow y$	$6.326e^{-10}$	$4.219e^{-14}$	0.5411
$x_2^3 \rightarrow y$	0.4103	0.0008	0.0006
$x_3 \rightarrow y$	0.9915	0.9447	$3.978e^{-05}$
$x_3^2 \rightarrow y$	0.8796	0.9771	0.6861
$x_3^3 \rightarrow y$	0.9672	0.9171	0.0096
$(x_1, x_2, x_3) \rightarrow y$	$<2.2e^{-16}$	$<2.2e^{-16}$	$<2.2e^{-16}$
$(x_1^2, x_2^3, x_3) \rightarrow y$	0.8430	0.4503	0.2679

3.2 Linearity Test for Interaction Effect

Simulation in linearity test for interaction effect were carried out with the same condition in simulation of quadratic and cubic effect. It was involved three data groups: first group, the data without multicolinearity, the second group the data with multicolinearity around 0.5 and the last group close to 1 (0.8). The equation of interaction effect is:

$$Y = X_1 + X_2 + (X_1X_2) + X_3 + \varepsilon \quad (17)$$

From the simulation, the elements of interaction effect can be analyzed with all indicators and the combination of indicators. If the significant value more than critical value as a result the indicators and the combination of indicators were put in the model

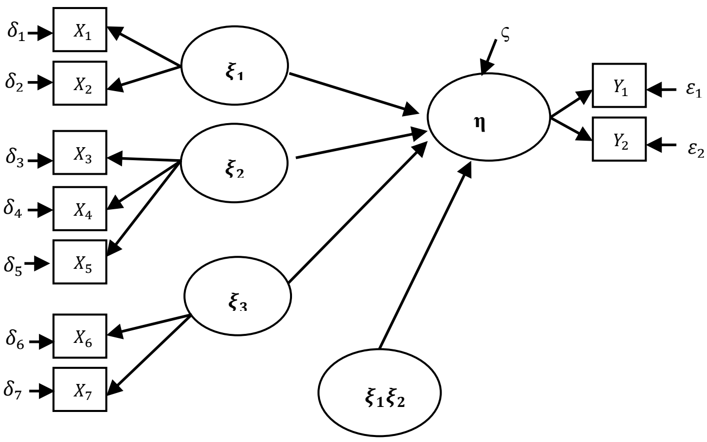

Figure 1 Structure of non-linear SEM
With interaction effect

Table 2. Result of non-linearity tes

Variable	**Uncorrelated**	**Multicolinearity**	
		0.5	**0.8**
$x_1 \rightarrow y$	0.1481	$9.46e^{-05}$	$5.532e^{-09}$
$x_2 \rightarrow y$	0.8337	$5.735e^{-06}$	$7.031e^{-11}$
$x_3 \rightarrow y$	0.8881	0.9507	$1.649e^{-06}$
$(x_1, x_2, x_3) \rightarrow$ y	$<2.2e^{-16}$	$<2.2e^{-16}$	$<2.2e^{-16}$
$(x_1, x_2, x_3, x_1x_2) \rightarrow y$	0.9257	0.6004	0.2813
$(x_1, x_2, x_3, x_1x_3) \rightarrow y$	$<2.2e^{-16}$	$<2.2e^{-16}$	$<2.2e^{-16}$
$(x_1, x_2, x_3, x_2x_3) \rightarrow y$	$<2.2e^{-16}$	$<2.2e^{-16}$	$<4.441e^{-16}$

4 Concluding Remarks

This study show that Lagrange Multiplier Test could be performed in non-linear SEM. The linearity test accomplish both in partial and simoultan.

Data with different multicolinearity will obtain the different analysis process in combination of quadratic elements and interaction elements. The data without multicolinerity, show that the elements of quadratic and interaction could be investigated from the significant value. The data with mulcticolinearity about 0.5 need more analysis in to determine the elements of the model. The data with high multicolinearity could not be examined with significant level. It means the data with high multicolinearity need more different analysis.

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