Enhancing Clustering Quality of Fuzzy Geographically Weighted Clustering using Ant Colony Optimization

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***Abstract*—Fuzzy Geographically Weighted Clustering (FGWC) is recognized as one of the most efficient methods for geo-demographic analysis problem. FGWC uses neighborhood effect to remedy the limitation of classical fuzzy clustering methods in terms of geographic factors. However, there are some drawbacks of FGWC such as sensitivity to cluster initialization phase that is required to overcome. Random initialization scheme of FGWC occasionally trapped into local optima which lead to poor clustering quality. In this paper, we propose a new hybrid approach of FGWC based on Ant Colony Optimization (ACO), namely FGWC-ACO, in which the initialization is performed and rigorously controlled by ACO fitness function. Based on the experimental simulation, the proposed method clearly outperforms the standard FGWC and offers a better geo-demographic clustering quality.**

I. Introduction

Geo-demographic clustering, as demonstrated by recent studies, succesfully provide a simple and effective utility of characterising population through a manageable set of groups in many fields of research [1]–[4]. This term is widely identified as the practice of grouping geographical areas according to the social and demographic characteristics of people who live within them [5]–[7]. The practice has been applied to many real world problems for the patterns extraction and implicit knowledge discovery wherein the fuzziness exist such as applied energy, government issues, transportation, customer management and acquisition, interactive marketing and educations [6], [8], [9].

For instance, Saarenpaa et al. used geo-demographic analysis to investigate the relationship between different social and demographic attibutes of the Finland regions and early hybrid electric vehicle adoption [8]. In Canada, Paez et al. reported the potential of analysis of geo-demographic to generate intelligence and identify business interest in transit smart cards [1]. Grekousis and Thomas performed a geo-demographic segmentation study to investigate the socio-economic diversity in a Greece prefecture [10].

Among some clustering methods used in this specific area, Fuzzy Geographically Weighted Clustering (FWGC) is considered as one of the most popular and efficient algorithm for geo-demographic problems [2], [3], [9]. The fuzzy partitions of geo-demographic clustering is more popular due to its membership flexibility lead to more promising result [6], [11]. Nonetheless, the FGWC has some limitations of clustering quality in terms of speed and clustering quality [9], [12]. Son et al was reported an approach to improve the limitation of FGWC in terms of computational time [12]. Some current studies also presented the proposal to ameliorate the FGWC limitations by integrating the spatial interaction model and intuitionistic fuzzy sets, while the initialization phase of FGWC was kept unchanged [9], [13].

Table I: Symbols and Descriptions

Symbol	Definition and Description
N	Number of data points
c	Number of clusters
d	Number of dimensions
μ_{ij}	The membership of data point j in cluster i
v_i	The center of cluster i
α	Weight of original membership
β	Weight of modified membership
ϵ	Tolerance threshold
m	Fuzziness parameter
a	Scaling variable of original cluster membership
b	Scaling variable of weighted cluster membership
PC	Partition Coefficient index
CE	Classification Entropy index
SC	Partition Index
S	Separation Index
IFV	A spatial cluster validity index

Ant Colony Optimization (ACO) is one of metaheuristic methods inspired by the foraging behaviour of real ants in the wild which lead to a general purpose optimisation technique [14]. ACO is also useful in solving several classes of discrete and continuous optimization problems [15].

In this research we aim to propose an integration of ACO based optimization and FGWC, namely FGWC-ACO to improve the geo-demographic clustering quality by overcoming the limitation of FGWC in its initialization phase. Our principal contribution in this paper is a hybrid method of geo-demographic clustering that provides better performance than the existing popular method, FGWC and its latest optimized versions [2]–[4], [6].

II. Related Works

A. Fuzzy Geographically Weighted Clustering (FGWC)

Fuzzy Geographically Weighted Clustering has ability to apply population and distance effects for analyzing a geo-demographic clustering in order to improve the standard Fuzzy C-Means algorithm. The influence of one area upon another is considered by FGWC as the product of the populations of the areas. The divisor implements a distance decay effect through the weighting factor. In every cycle of fuzzy clustering, the adjusted cluster membership for the FGWC algorithm is calculated using following formulas:

$$\mu_i' = \alpha\mu_i + \beta\frac{1}{A}\sum_{j}^{n} w_{ij}\mu_j \tag{1}$$

Where μ_i' is the new cluster membership of area i and μ_i is the old cluster membership of area i. The w_{ij} is the weight measuring the amount of interaction between a pair of areas. The weight is decided by distance between area centroids or the length of common boundary between areas, or both. The A parameter is determined to ensure that the average of weighted membership values is still in the range of 0 and 1. The and are respectively weights to old membership and the mean of membership values of surrounding areas and are defined as as follows:

$$\alpha + \beta = 1 \tag{2}$$

The parameters α and β are the scaling variables that influence the ratio of the original membership and the weighted (calculated) membership, respectively [6]

$$w_{ij} = \frac{(m_i m_j)^b}{d_{ij}^a} \tag{3}$$

Where $m_i m_j$ are the population of areas i and j respectively, d_{ij} is the distance between i and j, then a and b are user definable parameters [6].

B. Ant Colony Optimization

Ant Colony Optimization (ACO), which is first proposed by Marco Dorigo in 1999 [16], is a metaheuristic technique for hard discrete optimization [14], [17], [18]. One of the problems studied by ethnologists was to understand how almost blind insects like ants could manage to establish shortest route paths from their colony to feeding sources and back. When searching for food, ants initially explore the area surrounding their nest in a random manner. As soon as an ant finds a food source, it evaluates quantity and quality of the food and carries some of the found food to the nest. During the return trip, the ant deposits a chemical pheromone trail on the ground. The quantity of pheromone deposited, which may depend on the quantity and quality of the food, will guide other ants to the food source [17].

One simple approach for applying a discrete optimization algorithm like ACO to a continuous-domain problem is to divide each dimension i of the search space into discretized intervals. It is trying to minimize the n-dimensional problem $f(x)$, where $x = [x_1, ..., x_n]$, and $x_i \in [x_{i,min}, x_{i,max}]$

$$x_{i,min} = b_{i1} < b_{i2} < ... < b_{i,B_i} = x_{i,max} \tag{4}$$

In the field of Ant Colony Optimization (ACO), models of collective intelligence of ants are transformed into useful optimization techniques that find applications. In this paper, the problem-solving paradigm of ACO is explicated. As we are not interested in simula- tion of ant colonies, but in the use of artificial ant colonies as an optimization tool, ACO will have some major differences with a real (natural) one:
(1) artificial ants will have some memory,
(2) they will not be completely blind,
(3) they will live in an environment where time is discrete [17].

III. The Proposed Methods

The basic idea of our work is utilizing the ACO algorithm to select the cluster centers automatically in the initialization phase of FGWC and ensure the selection is optimized by evaluating the ACO fitness function. The basic objective function which will be minimized is:

$$J_{FGWC}(U,V;X) = \sum_{i=1}^{c}\sum_{k=1}^{n} u_{ik}^m |v_i - x_k|^2 \rightarrow min \tag{5}$$

where m is a weighting exponent which determines the fuzziness of the clusters, u_{ij} is an element of partition matrix, v_i is a cluster center and x_k is a data point. We can determine the cluster center using the following formula:

$$v_i = \frac{\sum_{k=1}^{n} u_{ik}^m x_k}{\sum_{k=1}^{n} u_{ik}^m} \tag{6}$$

On the other hand, the membership matrix of fuzzy cluster before geographical modification can be calculated as follow:

$$u_{ik} = \frac{1}{\sum_{j=1}^{c}(\frac{|v_i - x_k|}{|v_j - x_k|})^{\frac{2}{m-1}}} \tag{7}$$

After being calculated, we modify the membership matrix using formulas 5. Inspired by popular method to optimize objective function in classical fuzzy clustering as reported by Runkler and Katz [19], we can reformulate the subsequent computation of cluster centers in formulas 6 and the membership matrix in formulas 7 and 5 into two different objective functions, respectively:

$$J_{FGWC}(V;X) = \sum_{i=1}^{c}\sum_{k=1}^{n} \frac{|v_i - x_k|^2}{(\sum_{j=1}^{c}(\frac{|v_i - x_k|}{|v_j - x_k|})^{\frac{2}{m-1}})^m} \rightarrow min \tag{8}$$

$$J_{FGWC}(U;X) = \sum_{i=1}^{c}\sum_{k=1}^{n} u_{ik}^m \left|\frac{\sum_{k=1}^{n} u_{ik}^m x_k}{\sum_{k=1}^{n} u_{ik}^m} - x_k\right|^2 \rightarrow min \tag{9}$$

For simplicity, the formula 8 is defined as FGWC-V and formula 9 is defined as FGWC-U. Based on the result reported by [19] and [20], it is necessary to distinguish the treatment for different data set due to its dimension. For a data set that contains n number of records and d number of variables (dimensions), there are at least two different treatment. If $n > d$, encoded the objective function using cluster centers is simple, and could better handle data sets that $n > d$. In other hand, if $n < d$, encoded the objective function using membership is simple, and could better handle data sets that $n < d$. In this proposed method we will distinguish the treatment for those different condition of data.

IV. Experiments

We conducted a set of experiments to evaluate the performance of the proposed FGWC-ACO algorithm on various clustering parameter settings.

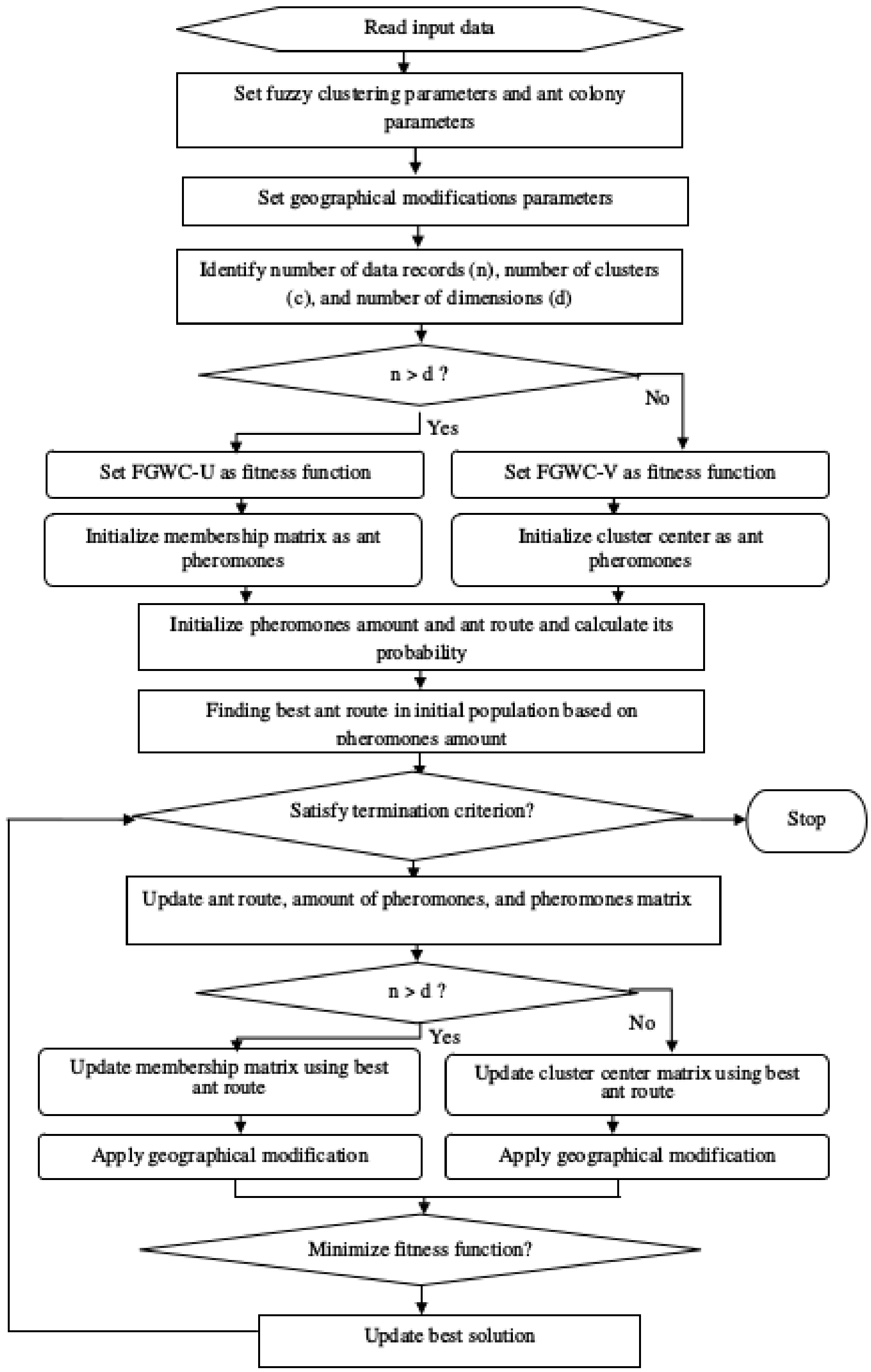

Figure 1: Flowchart of Proposed **FGWC-ACO** algorithm.

Dataset. We simulated the evaluation on publicly available dataset of Indonesia Population Census 2010 [4] contains demographic and economic variables at province level. It consists of 110 characteristics on educational attainment, ethnicity and language, housing details, population size and structure, household characteristics, marriage and divorce, economic activity, births, deaths, etc.

Comparison Methods. We compared the performance of our proposed methods againts the original FGWC [6], as well as its optimized versions which utilize Particle Swarm Optimization (FGWC-PSO) [4] and Artificial Bee Colony (FGWC-ABC) [2], [3] at the same environment and datasets.

Environment. All of the experiments were performed on the same machine with Intel Core i5-3210M processors @2.50GHz, 4.0GB RAM and Windows 7 64bit operating system. The execution times of those techniques are not considered, since execution times range less than a half of minute on the above machine.

Parameter Settings. We use the same parameters for all methods: α = 0.5, β = 0.5, a = 1, b = 1, and ϵ = 1e-6. We evaluate the robustness of all methods in various different m = [1.5, 2.0, 2.5, 3.0] anc c = [2-10].

Evaluation Metrics. The objective of this proposed method is clustering quality, which can be measured by Partition Coefficient (PC), Classification Entropy (CE), Partition Index (SC), Separation Index (S), and IFV index. Those measurement are usually used to measure the performance of clustering algorithms [10], [21]–[23], and are used as evaluation metrics in this work.

Those evaluation metrics are defined as follows [21]:

$$CE = -\frac{1}{N}\sum_{i=1}^{c}\sum_{j=1}^{N}\mu_{ij}log_a(\mu_{ij}) \tag{10}$$

$$SC = \sum_{i=1}^{c}\frac{\sum_{j=1}^{N}(\mu_{ij})^m|x_j - v_i|^2}{N_i\sum_{k=1}^{c}|v_k - v_i|^2} \tag{11}$$

$$S = \frac{\sum_{i=1}^{c}\sum_{j=1}^{N}(\mu_{ij})^2|x_j - v_i|^2}{Nmin_{i,k}|v_k - v_i|^2} \tag{12}$$

The lower values of those CE, SC, and S indices indicate the better clustering quality. On the other hand, the higher values of PC and IFV indices imply the better quality of resulted clusters, which are clearly derived from the definition as follows [21], [22]:

$$PC = \frac{1}{N}\sum_{i=1}^{c}\sum_{j=1}^{N}\mu_{ij}^2 \tag{13}$$

$$IFV = \frac{1}{c}\sum_{j=1}^{c}(\frac{1}{N}\sum_{k=1}^{N}\mu_{kj}^2[log_2 c - \frac{1}{N}\sum_{k=1}^{N}log_2\mu_{kj}]^2)\frac{SD_{max}}{\overline{\sigma_D}} \tag{14}$$

Where SD_{max} and $\overline{\sigma_D}$ can be computed by:

$$SD_{max} = max_{k\neq j}|v_k - v_j|^2 \tag{15}$$

$$\overline{\sigma_D} = \frac{1}{c}\sum_{j=1}^{c}(\frac{1}{N}\sum_{k=1}^{N}|x_k - v_j|^2) \tag{16}$$

V. Results

Here we evaluate the average clustering quality index values of the proposed methods FGWC-ACO against other competing methods (FGWC, FGWC-PSO, and FGWC-ABC) following by the number of clusters and the fuzziness value.

As illustrated in Table II that FGWC-ACO are outperformed the original FGWC and its current optimized version (FGWC-PSO and FGWC-ABC). The best value in each number of cluster are displayed in bold font and gray background. The higher values of PC and IFV indicate the better clustering quality. Those values are relatively decreased to the number of clusters. When fuzziness equals to 3.0 FGWC-ACO perform better than others. Similarly, when fuzziness value equals to 2.0 FGWC-ACO successfully improved the geo-demographic clustering quality of the original FGWC.

Regarding the CE, SC, and S indices, the lower values represent the better quality of resulted clusters. It is also clear that in this terms, FGWC-ACO reach the better geo-demographic clustering results. Overall, among 90 different settings in Table II, almost all of them are depicted the superiority of the proposed FGWC-ACO and leave 3 instances to other optimized methods. The proposed FGWC-ACO always successfully improved the FGWC quality in all different settings.

In the various fuzziness values (1.5, 2.0, 2.5, and 3.0) as displayed in Figure 2, it is clear that FGWC-ACO can reach better geo-demographic clustering quality compared to the original FGWC and its optimized counterparts. Besides, we can also implied from Figure 2 and Table II that the average IFV values of FGWC-ACO seem to give stable improvement through various fuzziness values againts other optimized methods.

VI. Conclusions

We aim to propose a new hybrid approach of FGWC based on Ant Colony Optimization (ACO) namely FGWC-ACO, in which the initialization is performed and controlled by the ACO fitness function. Based on the extensive experimental simulation, the proposed method clearly outperforms the standard FGWC and successfully provides a better geo-demographic clustering quality. Evaluated on various parameter settings, the proposed method also outperformed the other optimization models for FGWC algorithm in the most current literature such as Particle Swarm Optimization and Artificial Bee Colony.

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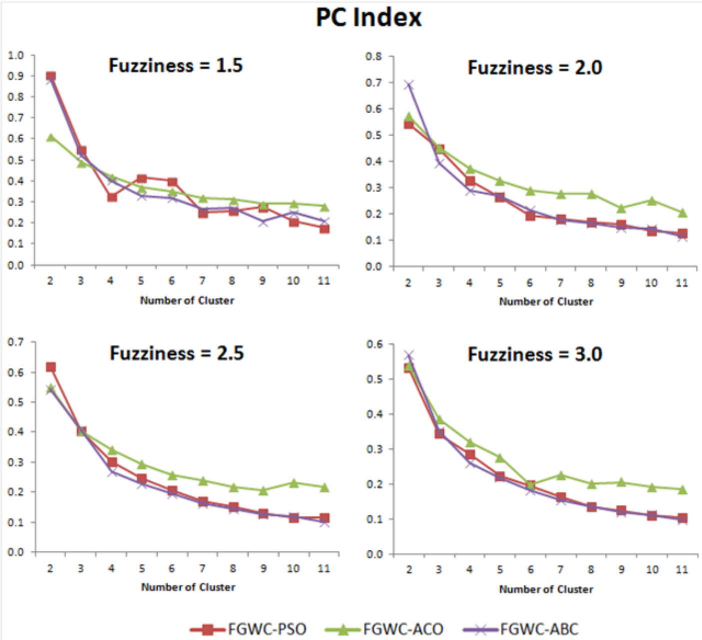
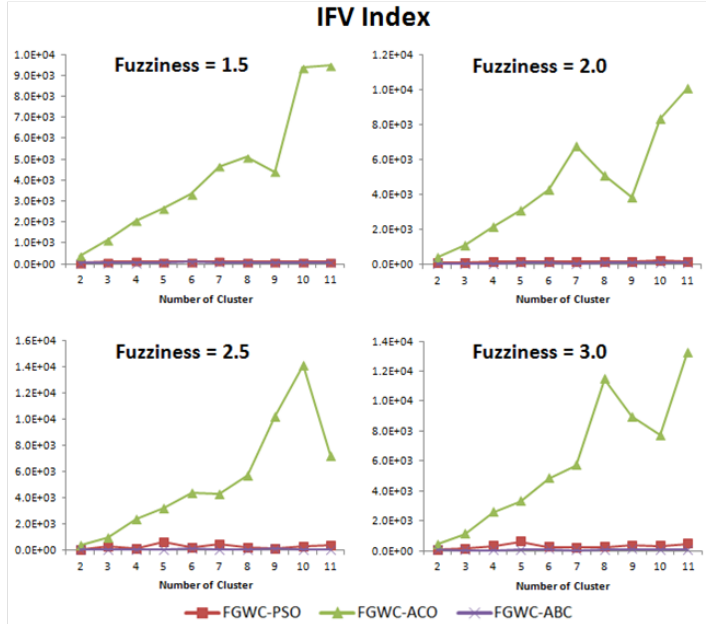
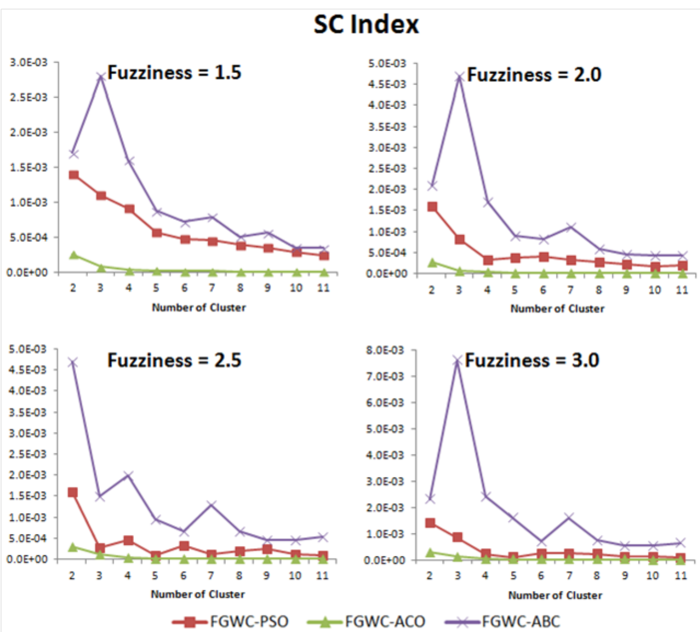


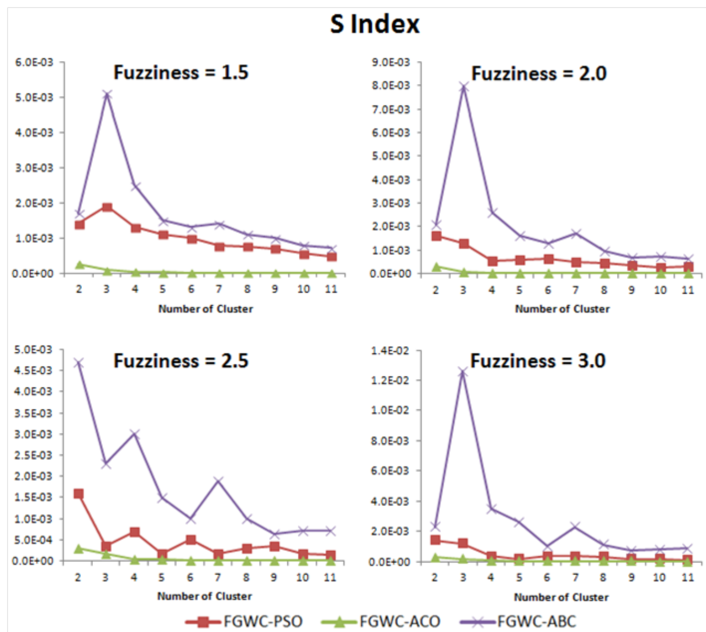
(a) **PC Index Evaluation** Higher is better.



(b) **IFV Index Evaluationp** Higher is better.



(c) **SC Index Evaluation** Lower is better.



(d) **S Index Evaluation** Lower is better.

Figure 2: **Clustering Quality Comparison.**

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Table II: **Clustering Quality Evaluation.**

	Fuzziness = 3.0				*Fuzziness = 2.0*			
c	FGWC	FGWC-PSO	FGWC-ABC	**FGWC-ACO**	FGWC	FGWC-PSO	FGWC-ABC	**FGWC-ACO**
PC Index. (higher is better)								
2	0.5	0.53	**0.567**	0.536	0.5	0.544	**0.695**	0.575
3	0.333	0.342	0.349	**0.384**	0.333	0.45	0.393	**0.453**
4	0.25	0.284	0.259	**0.318**	0.25	0.328	0.29	**0.374**
5	0.2	0.221	0.217	**0.274**	0.2	0.266	0.27	**0.326**
6	0.167	0.195	0.181	**0.196**	0.167	0.193	0.213	**0.290**
7	0.143	0.161	0.152	**0.224**	0.143	0.182	0.177	**0.277**
8	0.125	0.134	0.135	**0.199**	0.125	0.169	0.165	**0.276**
9	0.111	0.123	0.119	**0.204**	0.111	0.16	0.15	**0.221**
10	0.1	0.109	0.111	**0.191**	0.1	0.135	0.144	**0.252**
CE Index. (lower is better)								
2	0.693	0.662	**0.625**	0.656	0.693	0.648	**0.482**	0.616
3	1.099	1.086	1.075	**1.025**	1.099	0.934	1.009	**0.923**
4	1.386	1.325	1.369	**1.265**	1.386	1.213	1.314	**1.169**
5	1.609	1.563	1.566	**1.448**	1.609	1.438	1.461	**1.346**
6	1.792	**1.714**	1.753	1.721	1.792	1.722	1.659	**1.501**
7	1.946	1.884	1.916	**1.730**	1.946	1.809	1.831	**1.593**
8	2.079	2.043	2.044	**1.857**	2.079	1.916	1.937	**1.653**
9	2.197	2.148	2.167	**1.913**	2.197	2.014	2.062	**1.842**
10	2.303	2.259	2.253	**2.014**	2.303	2.149	2.111	**1.813**
SC Index. (lower is better)								
2	8.5.E+13	1.4.E-03	2.3.E-03	**2.9.E-04**	3.7.E+13	1.6.E-03	2.1.E-03	**2.9.E-04**
3	5.6.E+13	8.5.E-04	7.6.E-03	**1.2.E-04**	1.8.E+13	8.3.E-04	4.7.E-03	**7.7.E-05**
4	2.8.E+13	2.2.E-04	2.4.E-03	**3.5.E-05**	7.7.E+12	3.3.E-04	1.7.E-03	**3.2.E-05**
5	1.0.E+13	1.0.E-04	1.6.E-03	**2.2.E-05**	6.1.E+12	3.7.E-04	9.0.E-04	**2.1.E-05**
6	2.1.E+13	2.5.E-04	7.0.E-04	**1.4.E-05**	6.1.E+12	4.1.E-04	8.2.E-04	**1.4.E-05**
7	4.9.E+12	2.4.E-04	1.6.E-03	**1.1.E-05**	1.2.E+12	3.3.E-04	1.1.E-03	**9.0.E-06**
8	1.4.E+13	2.2.E-04	7.5.E-04	**5.2.E-06**	5.2.E+12	2.8.E-04	5.8.E-04	**7.6.E-06**
9	1.6.E+12	1.2.E-04	5.4.E-04	**4.9.E-06**	6.8.E+11	2.2.E-04	4.6.E-04	**9.0.E-06**
10	6.1.E+12	1.2.E-04	5.3.E-04	**3.5.E-06**	1.3.E+12	1.7.E-04	4.2.E-04	**3.7.E-06**
S Index. (lower is better)								
2	8.5.E+13	1.4.E-03	2.3.E-03	**2.9.E-04**	3.7.E+13	1.6.E-03	2.1.E-03	**2.9.E-04**
3	9.3.E+13	1.2.E-03	1.3.E-02	**1.8.E-04**	3.0.E+13	1.3.E-03	8.0.E-03	**8.9.E-05**
4	4.2.E+13	3.5.E-04	3.5.E-03	**4.4.E-05**	1.2.E+13	5.6.E-04	2.6.E-03	**4.0.E-05**
5	1.7.E+13	1.6.E-04	2.6.E-03	**3.1.E-05**	9.8.E+12	6.0.E-04	1.6.E-03	**2.8.E-05**
6	2.9.E+13	3.6.E-04	1.0.E-03	**1.9.E-05**	8.8.E+12	6.2.E-04	1.3.E-03	**2.2.E-05**
7	7.9.E+12	3.5.E-04	2.3.E-03	**1.4.E-05**	2.0.E+12	4.9.E-04	1.7.E-03	**1.2.E-05**
8	1.9.E+13	3.0.E-04	1.1.E-03	**6.8.E-06**	7.0.E+12	4.4.E-04	9.6.E-04	**1.1.E-05**
9	2.5.E+12	1.6.E-04	7.3.E-04	**7.2.E-06**	1.1.E+12	3.5.E-04	7.0.E-04	**1.2.E-05**
10	8.6.E+12	1.6.E-04	7.9.E-04	**4.8.E-06**	1.9.E+12	2.5.E-04	7.2.E-04	**4.9.E-06**
IFV Index. (higher is better)								
2	3.7.E-12	7.5.E+01	4.3.E+01	**4.2.E+02**	8.4.E-12	6.8.E+01	3.8.E+01	**3.9.E+02**
3	7.4.E-12	1.2.E+02	1.8.E+01	**1.1.E+03**	2.2.E-11	9.6.E+01	2.6.E+01	**1.1.E+03**
4	1.5.E-11	3.3.E+02	4.1.E+01	**2.6.E+03**	5.3.E-11	1.7.E+02	5.3.E+01	**2.2.E+03**
5	2.1.E-11	6.1.E+02	4.9.E+01	**3.3.E+03**	3.6.E-11	1.6.E+02	7.8.E+01	**3.1.E+03**
6	1.1.E-11	2.3.E+02	9.5.E+01	**4.8.E+03**	4.0.E-11	1.5.E+02	8.5.E+01	**4.3.E+03**
7	2.6.E-11	2.2.E+02	4.0.E+01	**5.7.E+03**	1.2.E-10	1.6.E+02	5.5.E+01	**6.8.E+03**
8	1.5.E-11	2.3.E+02	6.2.E+01	**1.2.E+04**	4.0.E-11	1.6.E+02	7.3.E+01	**5.1.E+03**
9	7.0.E-11	3.5.E+02	8.9.E+01	**9.0.E+03**	1.8.E-10	1.7.E+02	9.5.E+01	**3.8.E+03**
10	2.1.E-11	3.1.E+02	6.2.E+01	**7.7.E+03**	1.0.E-10	2.0.E+02	7.2.E+01	**8.3.E+03**

Imaging Graph., vol. 30, no. 1, Jan 2006.

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