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Small Area Estimation of Unemployment Rate Based on Unit Level Model with First Order Autoregressive Time Effects

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SUMMARY

Indonesian labor force participation data are collected by Sakernas (National Labor Force Survey). The main purpose of Sakernas is to obtain information about unemployment rate and its changes over time. The quarterly survey is designed only for estimating the parameters at provincial level. Also, the official quarterly unemployment rate is estimated based on only cross-sectional method despite the fact that the data are collected under rotating panel survey. The research at hand is aimed to estimate a quarterly unemployment rate at district level. There are three related issues, namely 1) insufficient sample size, 2) complication of panel rotation survey on parameter estimation, and 3) gaps in the parameter estimation. Related to the situation, small area estimation (SAE) model can be used to solve these issues. To solve these issues, two SAE models were proposed, in which the strength of information was borrowed over time by using panel data sets. The first model is a slightly modification of the Rao-Yu model suitable for analysis at unit level and the second model is very close to the Rao-Yu model. An emperical best linear unbiased predictor (EBLUP) based on the proposed model has been obtained. Simulation study shows that the estimation of the proposed model is better than the Rao-Yu model. The estimation of unemployment rate based on the proposed model was not affected by the direct estimation.

Keywords and phrases: EBLUP, Rao-Yu model, rotating panel survey, Sakernas, area unit level.

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1 Introduction

In many countries, including Indonesia, a rotating panel survey is used to collect data on labor force participation. The distinguishing feature of this survey compared to the conventional panel survey is that the respondents (or households) are divided into some parts called rotation groups, where each group is a subsample of the overall sample. In each periode one rotation group is introduced into the survey. Households in a rotation group are interviewed during some periods, then they are dropped at certain periods and substituted by households from a new rotation group. Each country has different rotation design.

In Indonesia, labor force participation data are collected by Sakernas (National Labor Force Survey). Sakernas has been regularly conducted since 1976 by BPS-Statistics Indonesia (Badan Pusat Statistik), but the rotating panel survey started from 2011. In the framework of the rotation panel, the total sample of census blocks are separated into four sample packages. Every package, at each census block is formed by 4 household groups. In every quarter the total sample consists of four groups coming from different packages. Sakernas rotation panel is designed on quarterly basis by maintaining 3/4 groups of the previous quarter and adding 1/4 new groups of the current quarter. An illustration of the Sakernas rotating panel design can be seen at Muchlisoh *et al.* [13] and for detail, see [1].

The main purpose of Sakernas is to obtain information about unemployment rate and its changes over time. The quarterly survey is designed only for estimating the parameters at provincial level. Estimation of parameters for district level only conducted in 3rd quarter in each year. It requires 3 times more than the number of samples for each quarter which implies the requirement of additional cost and time. Also, the official quarterly unemployment rate is estimated based on only cross-sectional method despite the fact that the data are collected under rotating panel survey. The research at hand is aimed to estimate a quarterly unemployment rate at district level.

There are three related issues, namely 1) insufficient sample size, 2) complication of panel rotation survey on parameter estimation, and 3) gaps in the parameter estimation. To solve these issues, it is necessary to study how to estimate parameter for district level based on rotating panel survey when sample size is insufficient. Related to the situation, small area estimation (SAE) model is an alternative that can be used to estimate the parameters of an area when the sample size in the area is too small to obtain an adequate precision which are estimated directly from survey [5]. One of SAE models for panel data is the Rao-Yu model [3],[4], an extension of the basic Fay-Herriot model [12] by adding a random area-time component which follows an autoregressive process order-1.

Some researchers have applied and modified the Rao-Yu model related to the constraints of autoregressive coefficient. You *et al.* [14] applied the Rao-Yu model to estimate the monthly unemployment rate for census metropolitan areas and census agglomerations in Canada using the Canadian Labour Force Survey. Esteban *et al.* [8] applied the Rao-Yu model to estimate poverty rates for the Spanish provinces by gender. Fay and Diallo [10] and Fay *et al.* [11] have modified the Rao-Yu model become a dynamic model. Diallo [9] have generalized the model Rao-Yu. The modification of the Rao-Yu model is still based on area level.

The Rao-Yu model did not specifically developed for panel rotation data, but estimating the unemployment rate at districts level using Sakernas panel rotation data in our previous study showed that the estimation based on the Rao-Yu model was better than direct estimation [13]. However, the basic area level model assumes that the sampling variances are known. In practice, the sampling variances are seldom known and must be replaced by an estimator [6]. Sakernas was designed based on three stages sampling design and to estimate the variances of three-stage sampling is not simple.

Related to the issue of estimation of sampling variances and three issues described at 4th paragraph, a SAE model was developed, in which the strength of information was borrowed over time by using panel data sets, but not specifically developed for panel rotation data. We closely followed the Rao-Yu model. In this article we proposed a slightly modification of the Rao-Yu model suitable for analysis at unit level. A simulation study was conducted and the quarterly unemployment rate at district level was also estimated.

The paper is structured as follows. Section 2 describes the Rao-Yu model and its modification, section 3 describes the proposed model, section 4 describes our simulation study, section 5 describes an application and section 6 describes conclusion.

2 The Rao-Yu Model and Its Modification

Let $\bar{y}_{it} = \sum_{j=1}^{n_{it}} y_{itj}$ be a direct survey estimator of i-th small area mean at time point t, say θ_{it} and $\bar{y}_{it}$ is assumed unbiased estimator for θ_{it}. The θ_{it}'s are related to:

$$\bar{y}_{it} = \theta_{it} + e_{it}. \tag{2.1}$$

The e_{it}'s are sampling errors that normally distributed with zero mean and known block diagonal covariance matrix $\boldsymbol{\Sigma}$. A vector of auxiliary variables, $\boldsymbol{x}_{it}$ related to θ_{it} is available such that:

$$\theta_{it} = \boldsymbol{x}'_{it}\boldsymbol{B} + v_i + u_{it}, \tag{2.2}$$

with $i = 1, \ldots, M$ and $t = 1, \ldots, T$. The $\boldsymbol{x}'_{it}$ is a vector of q fixed auxiliary population variables for i-th small area time t and the $\boldsymbol{B}$ is a vector of regression coefficients. Assumed $v_i \sim iidN(0, \sigma_v^2)$ is a random effect for i-th small area and u_{it} is a random effect for i-th small area at time point t. The u_{it}'s are assumed to follow a first order autoregressive process within each area i. The Rao-Yu model is combining part of model (2.1) and (2.2):

$$\begin{aligned} \bar{y}_{it} &= \boldsymbol{x}'_{it}\boldsymbol{B} + v_i + u_{it} + e_{it}, \\ u_{it} &= \rho u_{i,t-1} + \varepsilon_{it}, \quad |\rho| < 1, \end{aligned} \tag{2.3}$$

with $\varepsilon_{it} \sim iidN(0, \sigma_\varepsilon^2)$. The v_i, u_{it} and e_{it} are assumed mutually independent. The condition $|\rho| < 1$ ensures stationarity of the series defined by (2.3) in order to obtain an autoregressive process of order-1.

Fay and Diallo [10] and Fay *et al.*[11] have modified the Rao-Yu model by removing the stationarity requirement and modifying the random effect terms. They call the model as dynamic model. Unlike the Rao-Yu model, which assumes stationarity of the u_{it} series, the dynamic model does not assume stationarity, ρ is not constrained. The model is defined as follows

$$\begin{aligned} \bar{y}_{it} &= \boldsymbol{x}'_{it}\boldsymbol{B} + \rho^{t-1}v_i^* + u_{it}^* + e_{it}, \\ u_{it}^* &= 1, \quad \text{for } t = 1, \\ u_{it}^* &= \rho u_{i,t-1}^* + \varepsilon_{it}, \quad \text{for } t > 1, \end{aligned} \tag{2.4}$$

where v_i^* is a random effect for i-th small area at $t = 1$, $v_i^* \sim iidN(0, \sigma_{v^*}^2)$ and $\varepsilon_{it} \sim iidN(0, \sigma_\varepsilon^2)$. When $\rho > 1$, the model corresponds to a divergent situation in which areas become progressively more disparate. However, by dropping the stationarity assumption, the dynamic model is more appropriate for a situation in which the disparity among area dissipates over time. When $\sigma_{v^*}^2 = \frac{\sigma_\varepsilon^2}{1-\rho^2}$ and $|\rho| < 1$, the dynamic model becomes equivalent to a Rao-Yu model with $\sigma_v^2 = 0$.

Diallo [9] obtained the general Rao-Yu model through dropping the stationarity assumption by assuming finite series. The general Rao-Yu model has the same model as in (2.3) but the time series part is defined as,

$$u_{it} = \rho u_{i,t-1} + \varepsilon_{it}, \quad \text{for } t \geq 1 \text{ and } u_{i0} = 0, \tag{2.5}$$

with $\varepsilon_{it} \sim iidN(0, \sigma_\varepsilon^2)$. The error e_{it}, v_i and ε_{it} are assumed to be independent of each other.

3 The Proposed Models

We proposed 2 models, say them as Model-1 and Model-2. Model-1 is a slightly modification of the Rao-Yu model suitable for analysis at the unit level. Let y_{itj} be j-th sample unit of i-th small area at time t and assume that unit specific auxiliary data $\boldsymbol{x}_{itj}$ are available for each population element j in i-th small area at time t. Model-1 is defined as,

$$\begin{aligned} y_{itj} &= \boldsymbol{x}'_{itj}\boldsymbol{B} + v_i + u_{it} + e_{itj}, \\ u_{it} &= \rho u_{i,t-1} + \varepsilon_{it}, \quad |\rho| < 1, \end{aligned} \tag{3.1}$$

with, $j = 1, 2, \ldots, n_{it}$ where n_{it} is number of sample unit of i-th small area at time t. Assume $v_i \sim iidN(0, \sigma_v^2)$ is a random effect for i-th small area and u_{it} is a random effect for i-th small area at time point t. The u_{it}'s are assumed to follow a first order autoregressive process within each area i, with $\varepsilon_{it} \sim iidN(0, \sigma_\varepsilon^2)$. The error e_{itj} are assumed $e_{itj} \sim iidN(0, \sigma_e^2)$ and the v_i, u_{it} and e_{itj} are mutually independent.

Model-2 is very close to the Rao-Yu model. Model-2 is defined the same as the model in (2.3), but the element of diagonal of $\boldsymbol{\Sigma}$ is replaced by $\hat{\sigma}_e^2/n_{it}$ and the other element replaced by 0. The $\hat{\sigma}_e^2$ is an estimator of σ_e^2 that derived from Model-1.

Similar with the Rao-Yu model, Model-1 is a special case of the general linear mixed model which cover many small area models. For each small area-i, the data (y_{itj}) can be arranged as $y_i = (y'_{i1}, \ldots, y'_{iT})'$ with $y_{it} = (y_{it1}, \ldots, y_{itn_{it}})'$, so that model (3.1) may be written, in matrix form, as

$$\mathbf{y}_i = \mathbf{X}_i\boldsymbol{B} + v_i\mathbf{1}_{n_i} + (\text{blockdiag}_{1\leq t\leq T}(\mathbf{1}_{n_{it}}))\mathbf{u}_i + \mathbf{e}_i, \tag{3.2}$$

where, $\mathbf{X}_i = (\mathbf{X}_{i1}, \ldots, \mathbf{X}_{iT})', \mathbf{X}_{it} = (\boldsymbol{x}_{it1}, \ldots, \boldsymbol{x}_{itn_{it}})', \boldsymbol{x}_{itj} = (1, x_{1_{itj}}, \ldots, x_{q_{itj}})'$. The random effect v_i is a scalar, $u_i = (u_{i1}, \ldots, u_{iT})'$ and $e_i = (e'_{i1}, \ldots, e'_{iT})'$ with $e_{it} = (e_{it1}, \ldots, e_{itn_{it}})'$ and $n_i = \sum_{t=1}^{T} n_{it}$. The $\mathbf{1}_{n_i}$ and $\mathbf{1}_{n_{it}}$ are n_i-vector of 1s and n_{it}-vector of 1s, respectively.

For balanced data, $n_{i1} = n_{i2} = \ldots = n_{iT}$, model (3.2) becomes:

$$\mathbf{y}_i = \mathbf{X}_i \boldsymbol{B} + v_i \mathbf{1}_{n_i} + (\mathbf{I}_T \otimes \mathbf{1}_{n_{it}})\mathbf{u}_i + \mathbf{e}_i, \tag{3.3}$$

with, $\mathbf{I}_T$ is the identity matrix of order T and $\otimes$ is the kronecker product. Define $\boldsymbol{Z}_{1i} = \mathbf{1}_{n_i}$ and $\boldsymbol{Z}_{2i} = (\mathbf{I}_T \otimes \mathbf{1}_{n_{it}})$, model (3.3) may be written as

$$\mathbf{y}_i = \mathbf{X}_i \boldsymbol{B} + \boldsymbol{Z}_{1i} v_i + \boldsymbol{Z}_{2i} \mathbf{u}_i + \mathbf{e}_i. \tag{3.4}$$

For all small area, model (3.1) becomes:

$$\begin{aligned} \mathbf{y} &= \mathbf{X}\boldsymbol{B} + (\mathbf{I}_m \otimes \boldsymbol{Z}_{1i})\mathbf{v} + (\mathbf{I}_m \otimes \boldsymbol{Z}_{2i})\mathbf{u} + \mathbf{e}, \\ \mathbf{y} &= \mathbf{X}\boldsymbol{B} + \boldsymbol{Z}_1 \mathbf{v} + \boldsymbol{Z}_2 \mathbf{u} + \mathbf{e}, \end{aligned} \tag{3.5}$$

with $\mathbf{y} = (\mathbf{y}'_1, \ldots, \mathbf{y}'_m)'$, $\mathbf{X} = (\mathbf{X}'_1, \ldots, \mathbf{X}'_m)'$, $\mathbf{v} = (v'_1, \ldots, v'_m)'$, $\mathbf{u} = (\mathbf{u}'_1, \ldots, \mathbf{u}'_m)'$, and $\mathbf{e} = (\mathbf{e}'_1, \ldots, \mathbf{e}'_m)'$. The $\mathbf{I}_m$ is the identity matrix of order m.
The stationarity assumption of serial u_{it} in model (2.3) or (3.1) leads to

$$E(\mathbf{u}_i) = \mathbf{0} \quad \text{and} \quad Cov(\mathbf{u}_i) = \mathbf{G}_{2i} = \sigma_\varepsilon^2 \mathbf{\Gamma}, \tag{3.6}$$

where, $\mathbf{\Gamma}$ is a $T \times T$ symmetric matrix with (j,k)-th elements defined by $\frac{\rho^{|j-k|}}{1-\rho^2}$, where $j = 1, \ldots, T$ and $k = 1, \ldots, T$. The independence assumption of vector $e_i = (e'_{i1}, \ldots, e'_{iT})'$ leads to

$$E(\mathbf{e}_i) = \mathbf{0} \quad \text{and} \quad Cov(\mathbf{e}_i) = \mathbf{R}_i = \sigma_e^2 \mathbf{I}_{n_i}, \tag{3.7}$$

The covariance matrix of model (3.1) have block diagonal form when arranged by area. For each small area-i, the structure of covariance matrix is

$$\begin{aligned} \mathbf{V}_i &= Cov(\boldsymbol{Z}_{1i} v_i) + Cov(\boldsymbol{Z}_{2i} \mathbf{u}_i) + Cov(\mathbf{e}_i) \\ &= \sigma_v^2 \boldsymbol{Z}_{1i} \boldsymbol{Z}'_{1i} + \sigma_\varepsilon^2 \boldsymbol{Z}_{2i} \mathbf{\Gamma} \boldsymbol{Z}'_{2i} + \sigma_e^2 \mathbf{I}_{n_i} \\ &= \sigma_v^2 \mathbf{1}_{n_i} \mathbf{1}'_{n_i} + \sigma_\varepsilon^2 (\mathbf{I}_T \otimes \mathbf{1}_{n_{it}}) \mathbf{\Gamma} (\mathbf{I}_T \otimes \mathbf{1}_{n_{it}})' + \sigma_e^2 \mathbf{I}_{n_i} \\ &= \sigma_v^2 \mathbf{J}_{n_i} + \sigma_\varepsilon^2 (\mathbf{\Gamma} \otimes \mathbf{J}_{n_{it}}) + \sigma_e^2 \mathbf{I}_{n_i}. \end{aligned} \tag{3.8}$$

The $\mathbf{J}_{n_i}$ and $\mathbf{J}_{n_{it}}$ are matrices of order $n_i \times n_i$ and $n_{it} \times n_{it}$ with elements 1, respectively. For all area, the structure of covariance matrix is

$$\mathbf{V} = \text{blockdiag}_{1 \le i \le m}(\mathbf{V}_i)$$

and if $n_1 = n_2 = \ldots = n_m$, the structure of covariance matrix becomes

$$\begin{aligned} \mathbf{V} &= \mathbf{I}_m \otimes \mathbf{V}_i \\ &= \mathbf{I}_m \otimes \left(\sigma_v^2 \mathbf{J}_{n_i} + \sigma_\varepsilon^2 (\mathbf{\Gamma} \otimes \mathbf{J}_{n_{it}}) + \sigma_e^2 \mathbf{I}_{n_i} \right). \end{aligned} \tag{3.9}$$

Henderson [2] obtained the general form of BLUP (Best Linear Unbiased Prediction) of $\mathbf{w}$ for the general linear mixed model,

$$\mathbf{y} = \mathbf{X}\boldsymbol{B} + \boldsymbol{Z}\mathbf{w} + \mathbf{e} \tag{3.10}$$

is

$$\tilde{\mathbf{w}} = \mathbf{G}\mathbf{Z}'\mathbf{V}^{-1}(\mathbf{y} - \boldsymbol{X}\tilde{\boldsymbol{B}}) \tag{3.11}$$

with, $\mathbf{V} = \boldsymbol{R} + \mathbf{Z}\mathbf{G}\mathbf{Z}'$, $Cov\begin{pmatrix}\mathbf{w}\\ \mathbf{e}\end{pmatrix} = \begin{pmatrix}\mathbf{G} & \mathbf{0}\\ \mathbf{0} & \mathbf{R}\end{pmatrix}$ and $\tilde{\boldsymbol{B}} = [\mathbf{X}'\mathbf{V}^{-1}\mathbf{X}]^{-1}[\mathbf{X}'\mathbf{V}^{-1}\mathbf{y}]$.

Model (3.5) may be written as the general linear mixed model (3.10), where $\mathbf{Z} = (\mathbf{Z}_1, \mathbf{Z}_2)$ and $\mathbf{w} = \begin{pmatrix}\mathbf{v}\\ \mathbf{u}\end{pmatrix}$, with $\mathbf{R} = \sigma_e^2\mathbf{I}_m \otimes \mathbf{I}_{n_i}$ and $\mathbf{G} = \begin{pmatrix}\sigma_v^2\mathbf{I}_m & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_m \otimes \sigma_\varepsilon^2\boldsymbol{\Gamma}\end{pmatrix}$, so that by definition (3.11) the BLUP of $\begin{pmatrix}\mathbf{v}\\ \mathbf{u}\end{pmatrix}$ is

$$\begin{pmatrix}\tilde{\mathbf{v}}\\ \tilde{\mathbf{u}}\end{pmatrix} = \begin{pmatrix}\sigma_v^2\mathbf{I}_m & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_m \otimes \sigma_\varepsilon^2\boldsymbol{\Gamma}\end{pmatrix}((\mathbf{I}_m \otimes \boldsymbol{Z}_{1i}), (\mathbf{I}_m \otimes \boldsymbol{Z}_{1i}))'\,(\mathbf{I}_m \otimes \mathbf{V}_i)^{-1}(\mathbf{y} - \boldsymbol{X}\tilde{\boldsymbol{B}}).$$

The $\begin{pmatrix}\tilde{\mathbf{v}}\\ \tilde{\mathbf{u}}\end{pmatrix}$ can be partitioned into $\tilde{\mathbf{v}}$ and $\tilde{\mathbf{u}}$, then

$$\begin{aligned}\tilde{\mathbf{v}} &= (\sigma_v^2\mathbf{I}_m \otimes \boldsymbol{Z}_{1i}'\mathbf{V}_i^{-1})[(\mathbf{y}_1', \ldots, \mathbf{y}_m')' - (\mathbf{X}_1', \ldots, \mathbf{X}_m')'\tilde{\boldsymbol{B}}]\\ \tilde{\mathbf{u}} &= (\mathbf{I}_m \otimes \sigma_\varepsilon^2\boldsymbol{\Gamma}\boldsymbol{Z}_{2i}'\mathbf{V}_i^{-1})[(\mathbf{y}_1', \ldots, \mathbf{y}_m')' - (\mathbf{X}_1', \ldots, \mathbf{X}_m')'\tilde{\boldsymbol{B}}],\end{aligned}$$

so that,

$$\begin{aligned}\tilde{v}_i &= \sigma_v^2\boldsymbol{Z}_{1i}'\mathbf{V}_i^{-1}(\mathbf{y}_i - \boldsymbol{X}_i\tilde{\boldsymbol{B}})\\ &= \sigma_v^2\mathbf{1}_{n_i}'[\sigma_v^2\mathbf{J}_{n_i} + \sigma_\varepsilon^2(\boldsymbol{\Gamma} \otimes \mathbf{J}_{n_{it}}) + \sigma_e^2\mathbf{I}_{n_i}]^{-1}(\mathbf{y}_i - \boldsymbol{X}_i\tilde{\boldsymbol{B}})\end{aligned} \tag{3.12}$$

and

$$\begin{aligned}\tilde{\mathbf{u}}_i &= \sigma_\varepsilon^2\boldsymbol{\Gamma}\boldsymbol{Z}_{2i}'\mathbf{V}_i^{-1}(\mathbf{y}_i - \boldsymbol{X}_i\tilde{\boldsymbol{B}})\\ &= \sigma_\varepsilon^2\boldsymbol{\Gamma}\boldsymbol{Z}_{2i}'[\sigma_v^2\mathbf{J}_{n_i} + \sigma_\varepsilon^2(\boldsymbol{\Gamma} \otimes \mathbf{J}_{n_{it}}) + \sigma_e^2\mathbf{I}_{n_i}]^{-1}(\mathbf{y}_i - \boldsymbol{X}_i\tilde{\boldsymbol{B}}).\end{aligned} \tag{3.13}$$

The EBLUP of $\mathbf{v}$ and $\mathbf{u}$ are obtained by replacing the component of variance, $\rho, \sigma_v^2, \sigma_\varepsilon^2$ and σ_e^2 with their estimators, say $\hat{\rho}, \hat{\sigma}_v^2, \hat{\sigma}_\varepsilon^2$ and $\hat{\sigma}_e^2$, respectively. By REML estimation, their estimators have not a close form. It's not easy to derive estimates of their parameters and also the estimation of parameters is subject to constrains, especially the constraints of ρ. For the Rao-Yu model, the case of unknown ρ in the AR(1) model is more difficult to handle [5], as well as the proposed model. The stationarity assumption of serial u_{it} makes it difficult to estmate the parameter ρ when it is close to 1 [9]. We have done a simulation study to estimate their four parameters simultaneously by REML using Fisher Scoring iteration method, but we were not successfully obtained the estimator because the information Fisher matrix was often singular. Therefore, in this paper we will only discuss about parameter estimation when the parameter ρ is assumed to be known. Consequently, the EBLUP of v_i and $\mathbf{u}_i$ are,

$$\hat{v}_{i(\rho)} = \hat{\sigma_v}^2 \mathbf{1}'_{n_i}[\hat{\sigma}_v^2 \mathbf{J}_{n_i} + \hat{\sigma}_\varepsilon^2(\mathbf{\Gamma} \otimes \mathbf{J}_{n_{it}}) + \hat{\sigma}_e^2 \mathbf{I}_{n_i}]^{-1}(\mathbf{y}_i - \boldsymbol{X}_i \hat{\boldsymbol{B}}), \tag{3.14}$$

$$\hat{\mathbf{u}}_{i(\rho)} = \hat{\sigma}_\varepsilon^2 \mathbf{\Gamma} \boldsymbol{Z}'_{2i}[\hat{\sigma}_v^2 \mathbf{J}_{n_i} + \hat{\sigma}_\varepsilon^2(\mathbf{\Gamma} \otimes \mathbf{J}_{n_{it}}) + \hat{\sigma}_e^2 \mathbf{I}_{n_i}]^{-1}(\mathbf{y}_i - \boldsymbol{X}_i \hat{\boldsymbol{B}}). \tag{3.15}$$

Under REML estimation with ρ is known, the log likelihood function associated with the model (3.5) is equal to

$$lnL(\Omega) = -\frac{1}{2} ln|\mathbf{V}_{(\Omega)}| - \frac{1}{2} ln|\mathbf{X}'\mathbf{V}_{(\Omega)}^{-1} X| - \frac{1}{2}(\mathbf{y}'\mathbf{P}\mathbf{y}), \tag{3.16}$$

where $\mathbf{P} = \mathbf{V}_{(\Omega)}^{-1} - \mathbf{V}_{(\Omega)}^{-1}\mathbf{X}(\mathbf{X}'\mathbf{V}_{(\Omega)}^{-1}\mathbf{X})^{-1}(\mathbf{X}'\mathbf{V}_{(\Omega)}^{-1})$ with $\Omega = (\sigma_v^2, \sigma_\varepsilon^2, \sigma_e^2)$ and $\mathbf{V}_{(\Omega)} = \mathbf{V}$ is defined as (3.9). The estimator of $\Omega = (\sigma_v^2, \sigma_\varepsilon^2, \sigma_e^2)$ is obtained iteratively using Fisher Scoring method as follows

$$\Omega^{(k+1)} = \Omega^{(k)} + (\mathbf{F}_{(\Omega^{(k)})})^{-1} \frac{\partial lnL(\Omega^{(k)})}{\partial \Omega}. \tag{3.17}$$

The partial derivative of the log likelihood function to each parameter is

$$\frac{\partial lnL(\Omega)}{\partial \Omega_p} = -\frac{1}{2} tr\left(\mathbf{P}\frac{\partial \mathbf{V}_{(\Omega)}}{\partial \Omega_p}\right) + \frac{1}{2}\mathbf{y}'\mathbf{P}\frac{\partial \mathbf{V}_{(\Omega)}}{\partial \Omega_p}\mathbf{P}\mathbf{y}, \tag{3.18}$$

and the (p, p')-th elemen of 3×3 REML information matrix, $\mathbf{F}_{(\Omega)}$ is defined by

$$\mathbf{F}_{pp'} = -E\left(\frac{\partial^2 lnL(\Omega)}{\partial \Omega_p \partial \Omega_{p'}}\right) = \frac{1}{2} tr\left(\mathbf{P}\frac{\partial \mathbf{V}_{(\Omega)}}{\partial \Omega_p}\mathbf{P}\frac{\partial \mathbf{V}_{(\Omega)}}{\partial \Omega_{p'}}\right). \tag{3.19}$$

At the convergence of iteration (3.17), the REML estimator of parameter Ω, $\hat{\Omega} = (\hat{\sigma}_v^2, \hat{\sigma}_\varepsilon^2, \hat{\sigma}_e^2)$ will be obtained.

For a finite-population model, the i-th small area mean at time point t, θ_{it} is

$$\begin{aligned} \theta_{it} &= \frac{1}{N_{it}}\left[\sum_{j \in s} y_{itj} + \sum_{j \in r} y_{itj}\right] \\ &= \frac{n_{it}}{N_{it}}(\bar{y}_{it}^s) + \left(1 - \frac{n_{it}}{N_{it}}\right)(\bar{y}_{it}^r) \\ &= f_{it}(\bar{y}_{it}^s) + (1 - f_{it})(\bar{y}_{it}^r), \end{aligned} \tag{3.20}$$

where, N_{it} is the number of population unit of i-th small area at time t and $f_{it} = n_{it}/N_{it}$ is a sampling fraction. The upperscript "s" denoted the sampled units (observed units) and "r" denoted the non-sampled units (unobserved units). The first term of the equation (3.20), $\bar{y}^s_{it}$ can be derived through a direct estimation from the sample. Follow Battese *et al.* [7], the second term of the equation (3.20), $\bar{y}^r_{it}$ be predicted through the model prediction,

$$\begin{aligned}\bar{y}^r_{it} &= \frac{1}{N_{it}-n_{it}}\left[\sum_{j\in r}(\mathbf{x}'_{itj}\tilde{\boldsymbol{B}}+\tilde{v}_i+\tilde{u}_{it})\right]\\ &= \frac{1}{N_{it}-n_{it}}\left[\left(\sum_{j=1}^{N_{it}}\mathbf{x}'_{itj}-\sum_{j=1}^{n_{it}}\mathbf{x}'_{itj}\right)\tilde{\boldsymbol{B}}+(N_{it}-n_{it})(\tilde{v}_i+\tilde{u}_{it})\right]\\ &= \frac{1}{N_{it}-n_{it}}\left[N_{it}(\bar{\mathbf{x}}^p_{it})'-n_{it}(\bar{\mathbf{x}}^s_{it})'\right]\tilde{\boldsymbol{B}}+(\tilde{v}_i+\tilde{u}_{it})\end{aligned}\tag{3.21}$$

with the result that BLUP for θ_{it} is

$$\begin{aligned}\tilde{\theta}_{it} &= f_{it}(\bar{y}^s_{it})+(1-f_{it})\frac{1}{N_{it}-n_{it}}\left[N_{it}(\bar{\mathbf{x}}^p_{it})'-n_{it}(\bar{\mathbf{x}}^s_{it})'\right]\tilde{\boldsymbol{B}}+(\tilde{v}_i+\tilde{u}_{it})\\ &= f_{it}(\bar{y}^s_{it})+\frac{1}{N_{it}}\left[N_{it}(\bar{\mathbf{x}}^p_{it})'-n_{it}(\bar{\mathbf{x}}^s_{it})'\right]\tilde{\boldsymbol{B}}+(1-f_{it})(\tilde{v}_i+\tilde{u}_{it})\\ &= f_{it}(\bar{y}^s_{it})+\left[(\bar{\mathbf{x}}^p_{it})'-f_{it}(\bar{\mathbf{x}}^s_{it})'\right]\tilde{\boldsymbol{B}}+(1-f_{it})(\tilde{v}_i+\tilde{u}_{it})\end{aligned}\tag{3.22}$$

and when the f_{it} close to 0, the BLUP for θ_{it} becomes

$$\tilde{\theta}_{it} = (\bar{\mathbf{x}}^p_{it})'\tilde{\boldsymbol{B}}+(\tilde{v}_i+\tilde{u}_{it}).\tag{3.23}$$

The upperscript p denoted the population units.

The EBLUP of θ_{it} when ρ is known, $\hat{\theta}_{it}(\rho)$ is obtained by replacing $\tilde{\boldsymbol{B}}, \tilde{v}_i$ and $\tilde{u}_{it}$ with $\hat{\boldsymbol{B}}, \hat{v}_i(\rho)$ and $\hat{u}_{it}(\rho)$, respectively.

4 Simulation study

The simulation study starts from generating finite population that is assumed to follow model (3.1), $M = 26, T = 13$ and $j = 1, 2, \ldots, N_{it}$ with $N_{it} = 300$. To study effect of ρ and σ^2_v, we used $\rho = 0.2, \rho = 0.5, \rho = 0.9$ and $\sigma^2_v = 2.5, \sigma^2_v = 5, \sigma^2_v = 10$ while $\sigma^2_\varepsilon = 0.5$ and $\sigma^2_e = 19.5$. The parameter of the i-th small area mean at time point t defined by $\theta_{it} = \frac{1}{N_{it}}\sum_{j=1}^{N_{it}} y_{itj}$ and the direct estimator for θ_{it} defined by $y_{it} = \frac{1}{n_{it}}\sum_{j=1}^{n_{it}} y_{itj}$.

The estimation of θ_{it} will be performed by the proposed models and Rao-Yu model. To study the efficiency and bias of estimation of the proposed models, we selected 4 sample units ($n_{it} = 4$) from each small area by simple random sampling. The sampling is done as much as 100 replications ($R = 100$). From each simulated sample, the estimator of direct estimation, proposed models and Rao-Yu model were computed. Simulated values of the bias, MSE and CV of any estimator, say $\hat{\theta}_{it}$, were computed as follows:

$$\text{Bias} = \frac{1}{MT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\frac{1}{R}\sum_{l=1}^{R}(\hat{\theta}_{it_l} - \theta_{it})\right),$$
$$\text{MSE} = \frac{1}{MT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\frac{1}{R}\sum_{l=1}^{R}(\hat{\theta}_{it_l} - \theta_{it})^2\right),$$
$$\text{CV} = \frac{1}{MT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\frac{\sqrt{\frac{1}{R}\sum_{l=1}^{R}(\hat{\theta}_{it_l} - \theta_{it})^2}}{\hat{\theta}_{it}}\right) \times 100.$$

The simulation results of 100 replications are shown in Table 1. From Table 1 we see that the proposed model produces bias, MSE and CV smaller than Rao-Yu model. The bias of Rao-Yu model getting smaller when the autoregressive coefficient is high. Otherwise the bias of the proposed model is not affected by the value of autoregressive coefficient. The MSE and CV of the proposed model and Rao-Yu model increases with increasing the value of autoregressive coefficient and the value of variance of random area effect.

Table 1: Simulation results of 100 replications

		Bias			**MSE**			**CV**		
					σ_v^2					
Estimation Method	ρ	2.5	5	10	2.5	5	10	2.5	5	10
Model-1	0.2	0.012	0.012	0.012	0.839	0.875	0.885	10.199	10.556	13.617
Model-2		0.011	0.011	0.011	0.856	0.893	0.903	10.296	10.655	13.738
Rao-Yu Model		0.032	0.032	0.032	1.413	1.464	1.474	14.005	14.296	18.293
Direct		0.011	0.011	0.011	4.739	4.739	4.739	26.346	26.558	34.665
Model-1	0.5	0.012	0.012	0.012	0.901	0.937	0.947	10.724	11.051	14.231
Model-2		0.011	0.011	0.011	0.916	0.953	0.963	10.795	11.121	14.331
Rao-Yu Model		0.034	0.034	0.034	1.474	1.524	1.534	14.337	14.619	18.713
Direct		0.011	0.011	0.011	4.739	4.739	4.739	26.348	26.536	34.666
Model-1	0.9	0.011	0.011	0.011	1.021	1.024	1.034	13.191	14.759	12.612
Model-2		0.011	0.011	0.011	1.035	1.037	1.048	13.275	14.835	12.688
Rao-Yu Model		0.015	0.015	0.017	1.648	1.654	1.666	17.492	22.592	16.794
Direct		0.011	0.011	0.011	4.839	4.839	4.838	30.885	31.411	29.829

5 Application

In a previous study we have applied the Rao-Yu model and the dynamic model to estimate the quarterly unemployment rate for district level using West Java Sakernas panel rotation data [13]. In the present study we applied the proposed Model-1 and Model-2 to estimate the quarterly unemployment rate for district level using the same data.

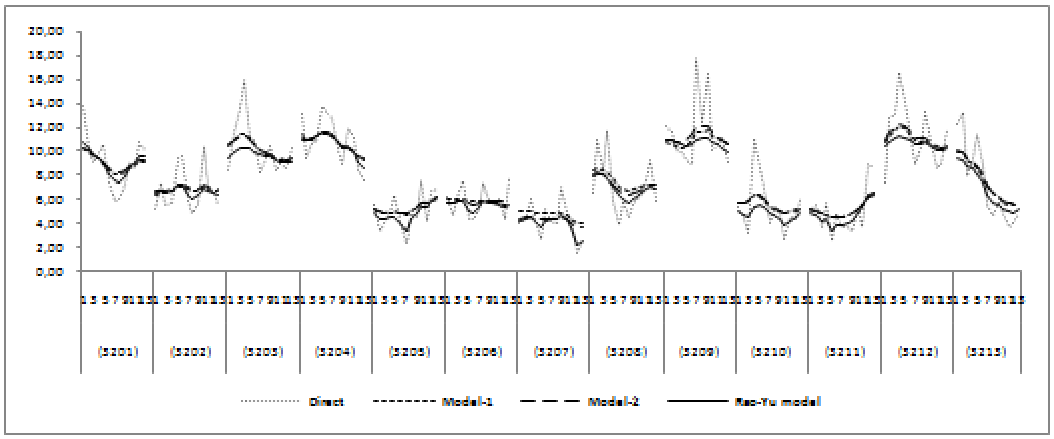
The total sample for West Java province consists of 400 census blocks (or about 4000 households). The sample was allocated proportionally to 26 districts. The total samples for each district is contained of about 8-21 census blocks. The sample of census block was only about 0.3 percent of 133,162. The total samples of census blocks are separated into four sample packages (1,2,3,4). Every package, at each census block is formed by 4 household rotation groups. So that in every quarter the total samples consists of four rotation groups coming from different packages. Each group in every census block consists of about 10 households. The total samples for each package contains of about 1-6 census blocks. We used 13 quarters of Sakernas panel rotation data from 2011 to 2014. The rotation groups are defined as a unit.

The proposed model assumes that unit specific auxiliary data are available for each population element for all time of observation. In Indonesia, the assumptions are not easy to be fulfilled. Consequently, in this study we use the same auxiliary data for all time. The auxiliary data come from The 2011 Village Potential Sensus.

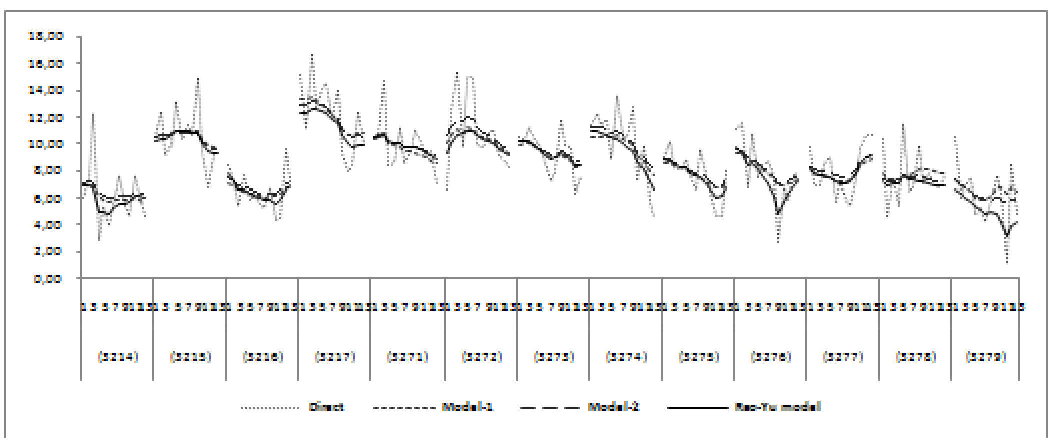
In the present study we assume that ρ is known. In practice, ρ is seldom known, therefore in this application study ρ is estimated by Rao-Yu model. Hereinafter, the estimator $\hat{\rho}$ is applied to Model-1 and Model-2. Based on Rao-Yu model with covariance matrix $\boldsymbol{\Sigma}$ is replaced by an estimator of sampling variance that derived from Taylor linearization, have been obtained $\hat{\rho} = 0.9$. Table 2 shows the estimator of variance components when $\rho = 0.9$. Figure 1 shows the comparison of estimation of quarterly unemployment rate at district level. Application studies which use the same autoregressive coefficients showed that the unit level proposed model (Model-1) is better in describing the variation between areas than area level models (Model-2 and Rao-Yu model). The estimation based on Rao-Yu model is still influenced by the direct estimation. The estimation based on Model-1 and Model-2 has similarities, but simulation studies showed that Model-1 is better than Model-2.

Table 2: **Estimator of variance components**

Model	$\hat{\sigma_v^2}$	$\hat{\sigma_\varepsilon^2}$	$\hat{\sigma_e^2}$
Model-1	2.58	0.57	19.54
Model-2	0.83	0.56	-
Rao-Yu Model	0.05	0.82	-



(a)



(b)

Figure 1: Comparison of estimator by district: (a) District code: 3201-3213, (b) District code: 3214-3279

6 Conclusion

The proposed model have several advantages compared to Rao-Yu model. The proposed model produces bias, MSE and CV smaller than Rao-Yu model, so that the estimation of the proposed model is better. Rao-Yu model assumes that the sampling variances are known, but in practice, the sampling variances are seldom to be known and must be replaced by an estimator. If sampling design is complex, it is not easy to calculate the estimator. Using the proposed model, no longer need to calculate the sampling variance because it is already estimated at once along with other variance components.

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